Class X Session 2023-24
Subject - Mathematics (Basic)
Sample Question Paper - 1

Time Allowed: 3 hours **Maximum Marks: 80**

General Instructions:

1. This Question Paper has 5 Sections A, B, C, D and E.
2. Section A has 20 MCQs carrying 1 mark each
3. Section B has 5 questions carrying 02 marks each.
4. Section C has 6 questions carrying 03 marks each.
5. Section D has 4 questions carrying 05 marks each.
6. Section E has 3 case based integrated units of assessment (04 marks each) with sub- parts of the values of 1, 1 and 2 marks each respectively.
7. All Questions are compulsory. However, an internal choice in 2 Qs of 5 marks, 2 Qs of 3 marks and 2 Questions of 2 marks has been provided. An internal choice has been provided in the 2marks questions of Section E
8. Draw neat figures wherever required. Take $\pi = \frac{22}{7}$ wherever required if not stated.

Section A

1. The HCF of 867 and 255 is **[1]**

a) 51 b) 35

c) 25 d) 55

2. The exponent of 2 in the prime factorisation of 144, is **[1]**

a) 4 b) 5

c) 6 d) 3

3. The discriminant of $4x^2 + 3x - 2 = 0$ is **[1]**

a) -23 b) 41

c) 39 d) -31

4. The graphs of the equations $2x + 3y - 2 = 0$ and $x - 2y - 8 = 0$ are two lines which are **[1]**

a) perpendicular to each other b) parallel

c) intersecting exactly at one point d) coincident

5. The roots of a quadratic equation are 5 and -2. Then, the equation is **[1]**

a) $x^2 - 3x + 10 = 0$ b) $x^2 - 3x - 10 = 0$

c) $x^2 + 3x + 10 = 0$ d) $x^2 + 3x - 10 = 0$

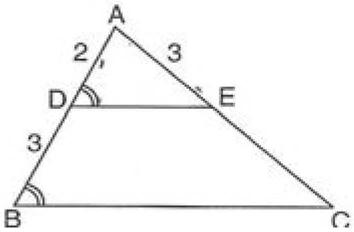
6. If the centroid of the triangle formed by the points (3, -5), (-7,4), (10, -k) is at the point (k, -1), then k = **[1]**

a) 2
b) 1
c) 4
d) 3

7. In $\triangle$ABC, it is given that AB = 9 cm, BC = 6 cm and CA = 7.5 cm. Also, $\triangle$DEF is given such that EF = 8 cm and $\triangle DEF \sim \triangle ABC$. Then, perimeter of $\triangle$DEF is **[1]**

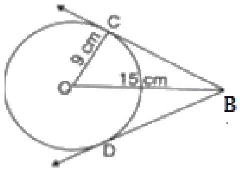
a) 30 cm
b) 22.5 cm
c) 27 cm
d) 25 cm

8. In the given figure if $\angle ADE = \angle ABC$, then CE is equal to **[1]**



a) 4.5
b) 3
c) 2
d) 5

9. In the adjoining figure, If OC = 9 cm and OB = 15 cm, then BC + BD is equal to **[1]**



a) 24 cm
b) 18 cm
c) 12 cm
d) 36 cm

10. If $(\cos\theta + \sec\theta) = \frac{5}{2}$ then $(\cos^2\theta + \sec^2\theta) = ?$ **[1]**

a) $\frac{33}{4}$
b) $\frac{21}{4}$
c) $\frac{17}{4}$
d) $\frac{29}{4}$

11. If the angle of elevation of a tower from a distance of 100 meters from its foot is 60°, then the height of the tower is **[1]**

a) $\frac{200}{\sqrt{3}}$ m
b) $50\sqrt{3}$ m
c) $100\sqrt{3}$ m
d) $\frac{100}{\sqrt{3}}$ m

12. $(\sec\theta + \cos\theta)(\sec\theta - \cos\theta) =$ **[1]**

a) $\tan^2\theta + \cos^2\theta$
b) $\tan^2\theta - \cos^2\theta$
c) $\tan^2\theta + \sin^2\theta$
d) $\tan^2\theta - \sin^2\theta$

13. The area of a sector of a circle with radius 6 cm if the angle of the sector is 60^o. **[1]**

a) $\frac{152}{7}$
b) $\frac{132}{7}$
c) $\frac{142}{7}$
d) $\frac{122}{7}$

14. A chord of a circle of radius 10 cm subtends a right angle at the centre. The area of the minor segments (given, π **[1]**

= 3.14) is

a) 32.5 cm^2

b) 34.5 cm^2

c) 30.5 cm^2

d) 28.5 cm^2

15. Raju bought a fish from a shop for his aquarium. The shop keeper takes out one fish from a tank containing 15 male fish and 18 female fish. The probability that the fish taken out is a male fish is **[1]**

a) $\frac{5}{11}$

b) $\frac{6}{11}$

c) $\frac{5}{12}$

d) $\frac{7}{11}$

16. The mean of the data when $\sum f_i d_i = 435$, $\sum f_i = 30$ and a = 47.5 is **[1]**

a) 47.5

b) 62

c) 30

d) 63

17. A solid is hemispherical at the bottom and conical (of same radius) above it. If the surface areas of the two parts are equal then the ratio of its radius and the slant height of the conical part is **[1]**

a) 4 : 1

b) 1 : 4

c) 1 : 2

d) 2 : 1

18. Consider the following frequency distribution: **[1]**

Class	0-5	6-11	12-17	18-23	24-29
Frequency	13	10	15	8	11

The upper limit of the median class is

a) 18.5

b) 17.5

c) 18

d) 17

19. **Assertion (A):** Distance of (5, 12) from y-axis is 5 units. **[1]**

Reason (R): Distance of point (h, k) from y-axis is always k units.

a) Both A and R are true and R is the correct explanation of A.

b) Both A and R are true but R is not the correct explanation of A.

c) A is true but R is false.

d) A is false but R is true.

20. **Assertion (A):** The H.C.F. of two numbers is 16 and their product is 3072. Then their L.C.M. = 162 **[1]**

Reason: If a, b are two positive integers, then H.C.F. × L.C.M. = a × b

a) Both A and R are true and R is the correct explanation of A.

b) Both A and R are true but R is not the correct explanation of A.

c) A is true but R is false.

d) A is false but R is true.

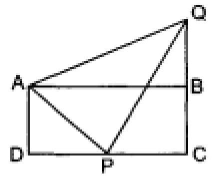
Section B

21. Is the pair of linear equation consistent/inconsistent? If consistent, obtain the solution graphically: 2x – 2y – 2 = 0; 4x – 4y – 5 = 0 **[2]**

22. In a $\triangle ABC$, AD is the bisector of $\angle A$, meeting side BC at D. If AD = 5.6 cm, BC = 6 cm and BD = 3.2 cm, find AC. **[2]**

OR

In the given figure, ABCD is a rectangle. P is mid-point of DC. If QB = 7 cm, AD = 9 cm and DC = 24 cm, then prove that $\angle APQ = 90°$.



23. The length of a tangent from a point A at distance 5 cm from the centre of the circle is 4 cm. Find the radius of the circle. **[2]**

24. Prove that: $\frac{\cot A-\cos A}{\cot A+\cos A}=\frac{cosec A-1}{cosec A+1}$ **[2]**

25. Find the area of a quadrant of a circle whose circumference is 22 cm. **[2]**

OR

Find the area of the segment of a circle of radius 14 cm, if the length of the corresponding arc APB is 22 cm.

Section C

26. Prove that $\frac{1}{\sqrt{2}}$ is irrational. **[3]**

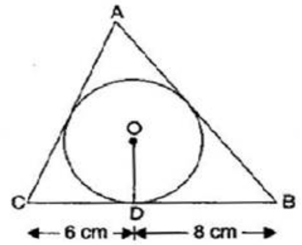
27. Write the family of quadratic polynomials having $-\frac{1}{4}$ and 1 as its zeros. **[3]**

28. Solve 2x + 3y = 11 and 2x – 4y = –24 and hence find the value of **m** for which y = mx + 3. **[3]**

OR

Solve graphically 2x - 3y + 13 = 0 and 3x - 2y + 12 = 0.

29. A triangle ABC is drawn to circumscribe a circle of radius 4 cm such that the segments BD and DC into which BC is divided by the point of contact D are of lengths 8 cm and 6 cm respectively (see figure). Find the sides AB and AC. **[3]**



30. In $\triangle ABC$, right angled at B, if $\tan A=\frac{1}{\sqrt{3}}$. Find the value of sin A cos C + cos A sin C. **[3]**

OR

If $\sec\theta=x+\frac{1}{4x}$, prove that: $\sec\theta+\tan\theta=2x$ or, $\frac{1}{2x}$

31. In a bag there are 44 identical cards with figure of circle or square on them. There are 24 circles, of which 9 are blue and rest are green and 20 squares of which 11 are blue and rest are green. One card is drawn from the bag at random. Find the probability that it has the figure of **[3]**
   i. square
   ii. green colour,
   iii. blue circle and
   iv. green square.

Section D

32. The sum of squares of two consecutive multiples of 7 is 637. Find the multiples. **[5]**

OR

If the roots of the quadratic equation (x - a) (x - b) + (x - b)(x - c) + (x - c)(x - a) = 0 are equal. Then show that a = b = c

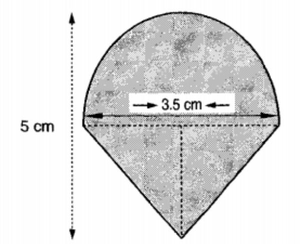
33. In a △ABC, XY is parallel to BC and it divides △ABC into two parts of equal area. Prove that $\frac{BX}{AB}=\frac{\sqrt{2}-1}{\sqrt{2}}$. **[5]**

34. A rocket is in the form of a circular cylinder closed at the lower end with a cone of the same radius attached to **[5]**

the top. The cylinder is of radius 2.5 m and height 21 m and the cone has the slant height 8 m. Calculate the total surface area of the rocket.

OR

Rasheed got a playing top (lattu) as his birthday present, which surprisingly had no colour on it. He wanted to colour it with his crayons. The top is shaped like a cone surmounted by a hemisphere. The entire top is 5 cm in height and the diameter of the top is 3.5 cm. Find the area he has to colour. (Take $\pi = \frac{22}{7}$).



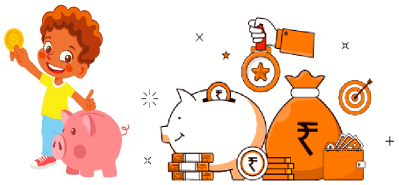
35. The following data gives the distribution of total monthly household expenditure of 200 families of a village. Find the modal monthly expenditure of the families. Also, find the mean monthly expenditure: **[5]**

Expenditure (in ₹)	**Frequency**
1000-1500	24
1500-2000	40
2000-2500	33
2500-3000	28
3000-3500	30
3500-4000	22
4000-4500	16
4500-5000	7

Section E

36. **Read the text carefully and answer the questions:** **[4]**

Saving money is a good habit and it should be inculcated in children from the beginning. Mrs. Pushpa brought a piggy bank for her child Akshar. He puts one five-rupee coin of his savings in the piggy bank on the first day. He increases his savings by one five-rupee coin daily.



(i) If the piggy bank can hold 190 coins of five rupees in all, find the number of days he can contribute to put the five-rupee coins into it

(ii) Find the total money he saved.

OR

How many coins are there in piggy bank on 15th day?

(iii) How much money Akshar saves in 10 days?

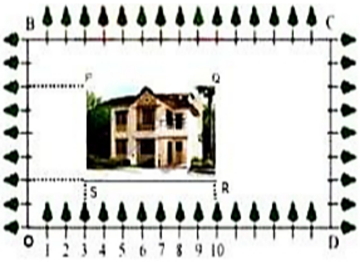
37. **Read the text carefully and answer the questions:** **[4]**

Using Cartesian Coordinates we mark a point on a graph by how far along and how far up it is.

The left-right (horizontal) direction is commonly called X-axis.

The up-down (vertical) direction is commonly called Y-axis.

In Green Park, New Delhi Suresh is having a rectangular plot ABCD as shown in the following figure. Sapling of Gulmohar is planted on the boundary at a distance of 1 m from each other. In the plot, Suresh builds his house in the rectangular area PQRS. In the remaining part of plot, Suresh wants to plant grass.



(i) Find the coordinates of the midpoints of the diagonal QS.

(ii) Find the length and breadth of rectangle PQRS?

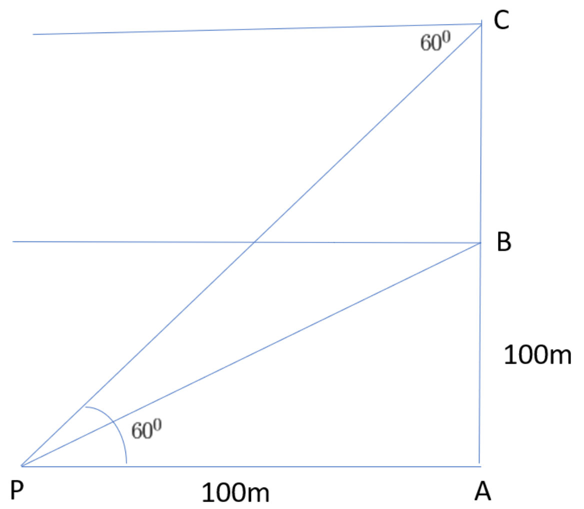
OR

Find the diagonal of rectangle.

(iii) Find Area of rectangle PQRS.

38. **Read the text carefully and answer the questions:** **[4]**

A hot air balloon is rising vertically from a point A on the ground which is at distance of 100m from a car parked at a point P on the ground. Amar, who is riding the balloon, observes that it took him 15 seconds to reach a point B which he estimated to be equal to the horizontal distance of his starting point from the car parked at P.



(i) Find the angle of depression from the balloon at a point B to the car at point P.

(ii) Find the speed of the balloon?

OR

After certain time Amar observes that the angle of depression is 60°. Find the vertical distance travelled by the balloon during this time.

(iii) Find the total time taken by the balloon to reach the point C from ground?

Solution

Section A

1. **(a)** 51

   Explanation: 867 = 255 $\times$ 3 + 102

   255 = 102 $\times$ 2 + 51

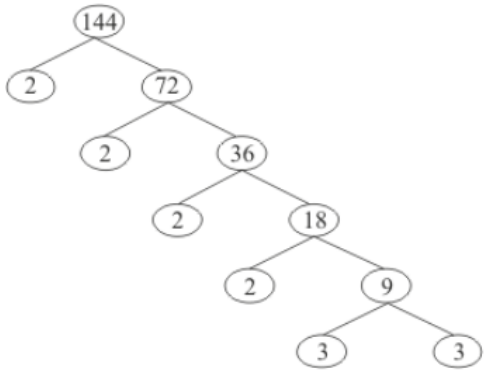
   102 = 51 $\times$ 2 + 0

   Hence, we got remainder as 0 , therefore HCF of (867 , 255) is 51

2. **(a)** 4

   Explanation:

   Using the factor tree for prime factorisation, we have:



   Therefore, 144 = $2 \times 2 \times 2 \times 2 \times 3 \times 3$

   $\Rightarrow 144 = 2^4 \times 3^2$

   Thus, the exponent of 2 in 144 is 4.

3.

   (b) 41

   Explanation: Here,

   a = 4, b = 3, c = -2

   Discriminant = b^2 - 4ac

   = $(3)^2$ - 4 $\times$ 4 $\times$ (-2)

   = 9 + 32 = 41

4.

   (c) intersecting exactly at one point

   Explanation: We have,

   2x + 3y – 2 = 0

   And, x – 2y – 8 = 0

   Here, a_1 = 2, b_1 = 3 and c_1 = – 2

   And, a_2 = 1, b_2 = – 2 and c_2 = – 8

   $\therefore \frac{a1}{a2} = \frac{2}{1}, \frac{b1}{b2} = \frac{3}{-2}$ and $\frac{c_1}{c_2} = \frac{-2}{-8} = \frac{1}{4}$

   Clearly, $\frac{a1}{a2} \neq \frac{b1}{b2}$

   Hence, the given system has a unique solution and the lines intersect exactly at one point.

5.

   (b) x^2 - 3x - 10 = 0

   Explanation: Sum of the roots = 5 + (-2) = 3, product of roots = 5 $\times$ (-2) = -10.

   $\therefore$ x^2 - (sum of roots) x + product of roots = 0.

   Hence, x^2 - 3x - 10 = 0.

6. **(a)** 2

Explanation: O(k, -1) is the centroid of triangle whose vertices are

A(3, -5), B(-7, 4), C(10, -k)

$\therefore k=\frac{x_1+x_2+x_3}{3}$

$\Rightarrow k=\frac{3-7+10}{3}=\frac{6}{3}=2$

7. **(a)** 30 cm

Explanation: $\triangle$DEF $\sim$ $\triangle$ABC

$\therefore \frac{\mathrm{DE}}{\mathrm{AB}}=\frac{\mathrm{EF}}{\mathrm{BC}}=\frac{\mathrm{DF}}{\mathrm{AC}}=\frac{\mathrm{DE}+\mathrm{EF}+\mathrm{DF}}{\mathrm{AB}+\mathrm{BC}+\mathrm{AC}}$

$\Rightarrow \frac{\mathrm{DE}}{9}=\frac{8}{6}=\frac{\mathrm{DF}}{7.5}$

$\frac{DE}{9}=\frac{8}{6} \Rightarrow DE=\frac{8\times 9}{6}=12\mathrm{cm}$

$\frac{\mathrm{DF}}{7.5}=\frac{8}{6} \Rightarrow \mathrm{DF}=\frac{7.5\times 8}{6}=10\mathrm{cm}$

Perimeter of $\triangle$DEF = DE + EF + DF

= 12 + 8 + 10 = 30 cm

8. **(a)** 4.5

Explanation: $\angle$ADE = $\angle$ABC and $\angle$DAE = $\angle$BAC. Hence ΔADE $\sim$ ΔABC(AA similarity)

hence the corresponding sides are in proportion

$\frac{AD}{AB}=\frac{AE}{Ac}$

$\Rightarrow \frac{2}{5}=\frac{3}{CE+3}$

$\Rightarrow$ CE = 4.5

9. **(a)** 24 cm

Explanation: Here $\angle$C = 90° [Angle between tangent and radius through the point of contact]

Now, in right angled triangle OBC,

$OB^2 = OC^2 + BC^2$

$\Rightarrow (9)^2 = (15)^2 + BC^2$

$\Rightarrow BC^2 = 225 - 81 = 144$

$\Rightarrow$BC = 12 cm

But BC = BD [Tangents from one point to a circle are equal]

Therefore, BD = 12 cm

Then BC + BD = 12 + 12 = 24 cm

10.

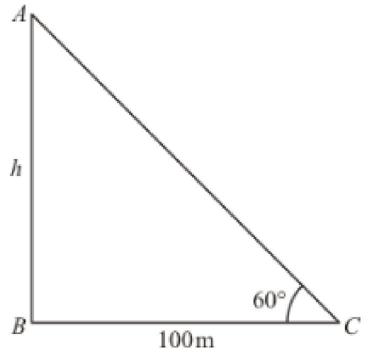
(c) $\frac{17}{4}$

Explanation: $(\cos\theta+\sec\theta)^2=\frac{25}{4} \Rightarrow \cos^2\theta+\sec^2\theta+2=\frac{25}{4} \Rightarrow \cos^2\theta+\sec^2\theta=\frac{25}{4}-2=\frac{17}{4}$

11.

(c) $100\sqrt{3}$ m

Explanation: Let AB be the height of tower is h meters



Given that: angle of elevation is 60^o from tower of foot and distance BC = 100 meters.

Here, we have to find the height of tower.

So we use trigonometric ratios.

In a triangle ABC,

$\Rightarrow$ tan C = $\frac{AB}{BC}$

$\Rightarrow$ tan 60° = $\frac{AB}{BC}$

$\Rightarrow \sqrt{3} = \frac{h}{100}$

$\Rightarrow$ h = 100 $\sqrt{3}$

12.

(c) $\tan^2\theta + \sin^2\theta$

Explanation: Given: $(\sec\theta + \cos\theta)(\sec\theta - \cos\theta)$

= $(\sec^2\theta - \cos^2\theta)$

= $(1 + \tan^2\theta - 1 + \sin^2\theta)$

= $(\tan^2\theta + \sin^2\theta)$

13.

(b) $\frac{132}{7}$

Explanation: Angle of the sector is 60°

Area of sector = $(\frac{\theta}{360^o}) \times \pi r^2$

$\therefore$ Area of the sector with angle 60° = $(\frac{60^o}{360^o}) \times \pi r^2$ cm^2

= $(\frac{36}{6})\pi$ cm^2

= $6 \times (\frac{22}{7})$ cm^2

= $\frac{132}{7}$ cm^2

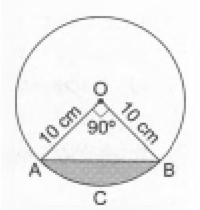
14.

(d) 28.5 cm^2

Explanation:

ar(minor segment A C B A)=ar(sector O A C B O) - ar(ΔOAB)

$= \left(\frac{\pi r^2\theta}{360} - \frac{1}{2} \times r \times r\right)$



$= \left(\frac{3.14\times10\times10\times90}{360} - \frac{1}{2} \times 10 \times 10\right) \text{cm}^2$

$= (78.5 - 50)\text{cm}^2 = 28.5\text{cm}^2$

15. **(a)** $\frac{5}{11}$

Explanation: Total number of fish = 15 + 18 = 33

Male fish = 15

Number of possible outcomes = 15

Number of total outcomes = 15 + 18 = 33

Required Probability = $\frac{15}{33} = \frac{5}{11}$

16.

(b) 62

Explanation: Mean = $(\overline{x}) = a + \frac{\sum f_i d_i}{\sum f_i}$

= 47.5 + $\frac{435}{30}$

= 47.5 + 14.5 = 62

17.

(c) 1 : 2

Explanation: $2\pi r^2 = \pi rl \Rightarrow \frac{r}{l} = \frac{1}{2}$

18.

(b) 17.5

Explanation: Given, classes are not continuous, so we make continuous by subtracting 0.5 from lower limit and adding 0.5 to upper limit of each class.

Class	Frequency	Cumulative frequency
-0.5-5.5	13	13
5.5-11.5	10	23
11.5-17.5	15	38
17.5-23.5	8	46
23.5-29.5	11	57

Here, $\frac{N}{2} = \frac{57}{2} = 28.5$, which lies in the interval 11.5 - 17.5.

Hence, the upper limit is 17.5.

19.

(c) A is true but R is false.

Explanation: Distance of (5, 12) from y-axis will be equal to x-coordinate to point. So, distance of (5, 12) from y-axis will be 5 units.

20.

(d) A is false but R is true.

Explanation: $\frac{3072}{16}$ = 192 $\neq$ 162

Section B

21. 2 x - 2 x - 2 = 0................(1)

4 x - 4 y - 5 = 0..................(2)

Here, $a_1 = 2, \quad b = -2, c_1 = -2$

$a_2 = 4, b_2 = -4, c_2 = -5$

We see that $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$

Hence, the lines represented by the equations(1) and (2) are parallel.

Therefore, equations (1) and (2) have no solution, i.e., the given pair of a linear equation is inconsistent.

22. If is is given that AB = 5.6 cm, BC = 6 cm and BD = 3.2 cm

In $\triangle ABC$, AD is the bisector of $\angle A$, meeting side BC at D

$\therefore \frac{AB}{AC} = \frac{BD}{DC}$

$\frac{5.6\text{cm}}{AC} = \frac{3.2\text{cm}}{2.8\text{cm}}$ [DC = BC - BD]

$AC = \frac{5.6 \times 2.8}{3.2}$ cm = 4.9

OR

According to question it is given that ABCD is a rectangle and p is the midpoint of DC.

$\therefore$ AD = BC = 9 cm

QC = BQ + BC = 7 + 9 = 16 cm

PC = $\frac{1}{2}$CD $\Rightarrow$ PC = 12 cm

In right $\triangle$PCQ using Pythagoras theorem

$PQ^2 = QC^2 + PC^2$

$PQ^2 = 16^2 + 12^2 = 400 \Rightarrow PQ = 20$ cm

In right $\triangle$ABQ using Pythagoras theorem

$AQ^2 = AB^2 + BQ^2 \Rightarrow AQ^2 = 24^2 + 7^2 = 625$

$\Rightarrow$ AQ = 25 cm

In right $\triangle$ADP using Pythagoras theorem

$AP^2 = AD^2 + DP^2 \Rightarrow AP^2 = 9^2 + 12^2$

$\Rightarrow AP^2 = 81 + 144$

$\Rightarrow AP^2 = 255$

AP = 15 cm

In $\triangle$APQ,

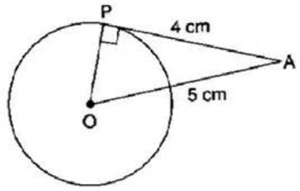
$AP^2 = 15^2 = 225$

$PQ^2 = 20^2 = 400 \Rightarrow AP^2 + PQ^2 = 625$

Also, $AQ^2 = 25^2 = 625 \Rightarrow AQ^2 = AP^2 + PQ^2$

$\therefore \triangle APQ$ is a right angled $\triangle$(using converse of BPT)

$\therefore \angle APQ = 90°$

23. 

We know that the tangent at any point of a circle is $\perp$ to the radius through the point of contact.

$\therefore \angle OPA = 90^o$

$\therefore OA^2 = OP^2 + AP^2$ [By Pythagoras theorem]

$\Rightarrow (5)^2 = (OP)^2 + (4)^2$

$\Rightarrow 25 = (OP)^2 + 16$

$\Rightarrow OP^2 = 9$

$\Rightarrow OP = 3$ cm

24. LHS = $\frac{\cot A - \cos A}{\cot A + \cos A}$

$= \frac{\frac{\cos A}{\sin A} - \cos A}{\frac{\cos A}{\sin A} + \cos A}$

$= \frac{\frac{\cos A - \sin A \cos A}{\sin A}}{\frac{\cos A + \sin A \cos A}{\sin A}}$

$= \frac{\cos A(1 - \sin A)}{\cos A(1 + \sin A)}$

$= \frac{1 - \sin A}{1 + \sin A}$

$= \frac{\frac{1}{\sin A} - 1}{\frac{1}{\sin A} + 1}$

$= \frac{cosec A - 1}{cosec A + 1}$ = RHS

25. Let the radius of the circle be r cm.

Then, circumference of the circle = 2πr cm

According to the question,

2πr = 22

$\Rightarrow 2 \times \frac{22}{7} \times \mathrm{r} = 22$

$\Rightarrow \mathrm{r} = \frac{22 \times 7}{2 \times 22} \Rightarrow \mathrm{r} = \frac{7}{2}\mathrm{cm}$

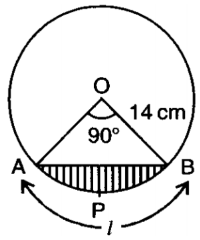
For a quadrant of a circle,

Area = $\frac{1}{4}\pi r^2$

$= \frac{1}{4} \times \frac{22}{7} \times \left(\frac{7}{2}\right)^2$

$= \frac{1}{4} \times \frac{22}{7} \times \frac{7}{2} \times \frac{7}{2} = \frac{77}{8}\mathrm{cm}^2$

OR



l = APB = 22 cm

$\frac{\theta}{180°} \times \frac{22}{7} \times 14 = 22\mathrm{cm}$

$\Rightarrow \quad \theta = 90°$

Area of the sector = $\frac{lr}{2} = \frac{22 \times 14}{2}$ = 154 cm^2

Area of triangle AOB= $\frac{1}{2} \times \mathrm{OA} \times \mathrm{OB} = \frac{1}{2} \times 14 \times 14 = 98\ \mathrm{cm}^2$

Area of the segment = (154 - 98) cm^2 = 56 cm^2

Section C

26. We can prove $\frac{1}{\sqrt{2}}$ irrational by contradiction.

Let us suppose that $\frac{1}{\sqrt{2}}$is rational.

It means we have some co-prime integers *a* and *b* ($b \neq 0$)

Such that

$\frac{1}{\sqrt{2}} = \frac{a}{b}$

$\Rightarrow \sqrt{2} = \frac{b}{a}$ ....(1)

R.H.S of (1) is rational but we know that is$\sqrt{2}$ irrational.

It is not possible which means our supposition is wrong.

Therefore,$\frac{1}{\sqrt{2}}$can not be rational.

Hence, it is irrational.

27. We know that, if x = a is a zero of a polynomial then x - a is a factor of quadratic polynomials.

Since $\frac{-1}{4}$ and 1 are zeros of polynomial.

Therefore $\left(x + \frac{1}{4}\right)$ (x - 1)

$= x^2 + \frac{1}{4}x - x - \frac{1}{4}$

$= x^2 + \frac{1}{4}x - \frac{4}{4}x - \frac{1}{4}$

$= x^2 + \frac{1-4}{4}x - \frac{1}{4}$

$= x^2 - \frac{3}{4}x - \frac{1}{4}$

Hence, the family of quadratic polynomials is f(x) = $k\left(x^2 - \frac{3}{4}x - \frac{1}{4}\right)$, where k is any non-zero real number.

28. The given pair of linear equations

2x + 3y = 11 ...... (1)

2x - 4y = -24 .... (2)

From equation (1), 3y = 11 - 2x

$\Rightarrow \quad y = \frac{11-2x}{3}$

Substituting this value of y in equation (2), we get

$2x - 4\left(\frac{11-2x}{3}\right) = -24$

$\Rightarrow 6x - 44 + 8x = -72$

$\Rightarrow 14x - 44 = -72$

$\Rightarrow 14x = 44 - 72$

$\Rightarrow 14x = -28$

$\Rightarrow x = -\frac{28}{14} = -2$

Substituting this value of x in equation (3), we get

$y = \frac{11-2(-2)}{3} = \frac{11+4}{3} = \frac{15}{3} = 5$

Verification, Substituting x = -2 and y = 5, we find that both the equations (1) and (2) are satisfied as shown below:

$2x + 3y = 2(-2) + 3(5) = -4 + 15 = 11$

$2x - 4y = 2(-2) - 4(5) = -4 - 20 = -24$

This verifies the solution,

Now, y = axe + 3

$\Rightarrow 5 = m(-2) + 3$

$\Rightarrow -2m = 5 - 3$

$\Rightarrow -2m = 2$

$\Rightarrow$ m $= \frac{2}{-2} = -1$

OR

2x - 3y + 13 = 0

3x - 2y + 12 = 0

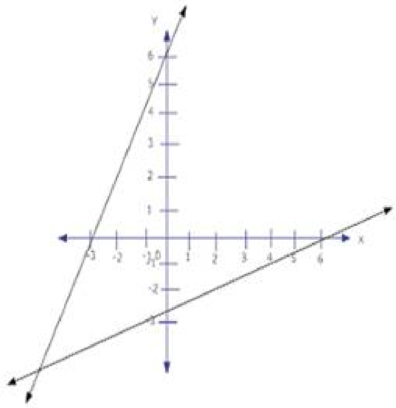
when $x = \frac{-13+3y}{2}$

x	6.5	5

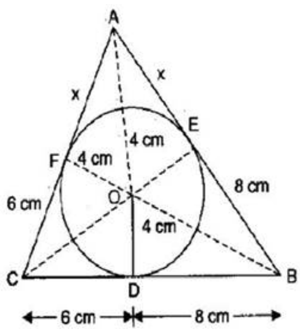
y	0	-1

when $y = \frac{3x+12}{2}$

x	0	-3
y	6	3



29. Join OE and OF. Also join OA, OB and OC.



Since BD = 8 cm

$\therefore$ BE = 8 cm

[Tangents from an external point to a circle are equal]

Since CD = 6 cm

$\therefore$ CF = 6 cm

[Tangents from an external point to a circle are equal]

Let AE = AF = x

Since OD = OE = OF = 4 cm [Radii of a circle are equal]

$\therefore$ Semi-perimeter of $\triangle$ ABC = $\frac{(x+6)(x+8)+(6+8)}{2} = \frac{(2x+28)}{2}$ = (x + 14)cm

$\therefore$ Area of $\triangle$ABC = $\sqrt{s(s-a)(s-b)\ (s-c)}$

= $\sqrt{(x+14)(x+14-14)(x+14-\overline{x+8})(x+14-\overline{x+6})}$

= $\sqrt{(x+14)(x)(8)(6)}$ cm ^{2}

Now, Area of ΔABC = Area of $\triangle$OBC + Area of $\triangle$OCA + Area of $\triangle$OAB

$\Rightarrow \sqrt{(x+14)(x)(8)(6)} = \frac{(6+8)4}{2} + \frac{(x+6)4}{2} + \frac{(x+8)4}{2}$

$\Rightarrow \sqrt{(x+14)(x)(8)(6)}$ = 28 + 2x + 12 + 2x + 16

$\Rightarrow \sqrt{(x+14)(x)(8)(6)}$ = 4x + 56

$\Rightarrow \sqrt{(x+14)(x)(8)(6)}$ =4(x + 14)

Squaring both sides,

(x + 14) (x) (6) (8) = 16$(x + 14)^2$

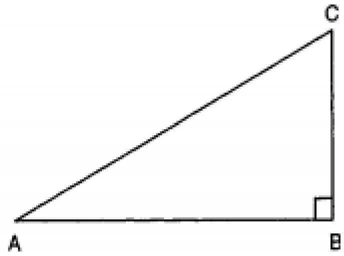
$\Rightarrow$ 3x = x + 14

$\Rightarrow$ 2x = 14

$\Rightarrow$ x = 7

$\therefore$ AB = x + 8 = 7 + 8 = 15 cm

And AC = x + 6= 7 + 6 = 13 cm

30. 

we have,

$\tan A = \frac{1}{\sqrt{3}} = tan 30^\circ$

$\therefore A = 30^\circ$

In $\triangle$ABC, we have

$\angle A + \angle B + \angle C = 180^\circ$

$\Rightarrow 30^\circ + 90^\circ + \angle C = 180^\circ$

$\Rightarrow 120^\circ + \angle C = 180^\circ$

$\Rightarrow \angle C = 180^\circ - 120^\circ = 60^\circ$

So,

sinA . cosC + cos A . sin C

$= \sin 30^\circ . \cos 60^\circ + \cos 30^\circ . \sin 60^\circ$

$= \frac{1}{2} \cdot \frac{1}{2} + \frac{\sqrt{3}}{2} \cdot \frac{\sqrt{3}}{2} = 1$

OR

By the given condition of question

$\sec\theta = x + \frac{1}{4x}$

$\therefore \quad \tan^2\theta = \sec^2\theta - 1$

$\Rightarrow \quad \tan^2\theta = \left(x + \frac{1}{4x}\right)^2 - 1 = x^2 + \frac{1}{16x^2} + \frac{1}{2} - 1 = x^2 + \frac{1}{16x^2} - \frac{1}{2} = \left(x - \frac{1}{4x}\right)^2$

$\Rightarrow \quad \tan\theta = \pm\left(x - \frac{1}{4x}\right)$

$\Rightarrow \quad \tan\theta = \left(x - \frac{1}{4x}\right)$ or, $\tan\theta = -\left(x - \frac{1}{4x}\right)$

CASE 1: When $\tan\theta = -\left(x - \frac{1}{4x}\right)$: In this case,

$\sec\theta + \tan\theta = x + \frac{1}{4x} + x - \frac{1}{4x} = 2x$

CASE 2: When $\theta = -\left(x - \frac{1}{4x}\right)$: In this case,

$\sec\theta + \tan\theta = \left(x + \frac{1}{4x}\right) - \left(x - \frac{1}{4x}\right) = \frac{2}{4x} = \frac{1}{2x}$

Hence, $\sec\theta + \tan\theta = 2x$ or , $\frac{1}{2x}$

31. Number of identical cards = 44

Out of 44 cards, one card can be drawn in 44 ways.

$\therefore$ Total number of elementary events = 44

Number of circles = 24

Number of blue circles = 9

$\therefore$ Number of green circles = 24 - 9 = 15

Number of squares = 20

Number of blue squares = 11

$\therefore$ Number of green squares = 20 - 11 = 9

i. Number of square = 20

$\therefore$ Favourable number of elementary events = 20

Hence, required probability = $\frac{20}{44} = \frac{5}{11}$

ii. Number of green figures = Number of green circles + Number of green square

=15 + 9 = 24

$\therefore$ Favourable number of elementary events = 24

Hence, required probability = $\frac{24}{44} = \frac{6}{11}$

iii. Number of blue circles = 9

$\therefore$ Favourable number of elementary events = 9

Hence, required probability = $\frac{9}{44}$

iv. Number of green squares = 9

$\therefore$ Favourable number of elementary events = 9

Hence, required probability = $\frac{9}{44}$.

Section D

32. According to the question, let the consecutive multiples of 7 be 7x and 7x +7

$(7x)^2 + (7x+7)^2 = 637$

or, $49x^2 + 49x^2 + 49 + 98x = 637$

or, $98x^2 + 98x - 588 = 0$

or, $x^2 + x - 6 = 0$

or, (x + 3)(x - 2) = 0

or, x = -3 ,2

Rejecting the value, x=2

Thus,the required multiples are, 14 and 21.

OR

Given,

(x - a)(x - b) + (x - b)(x - c) + (x - c)(x - a) = 0

$\Rightarrow x^2 - ax - bx + ab + x^2 - bx - cx + bc + x^2 - cx - ax + ac = 0$

$\Rightarrow 3x^2 - 2ax - 2bx - 2cx + ab + bc + ca = 0$

For equal roots $B^2 - 4AC = 0$

or, $\{-2(a+b+c)\}^2 = 4 \times 3(ab + bc + ca)$

or, $4(a+b+c)^2 - 12(ab+bc+ca) = 0$

or, $a^2 + b^2 + c^2 + 2ab + 2bc + 2ac - 3ab - 3bc - 3ac = 0$

or, $\frac{1}{2}\left[2a^2 + 2b^2 + 2c^2 - 2ab - 2ac - 2bc\right] = 0$

or, $\frac{1}{2}\left[\left(a^2 + b^2 - 2ab\right) + \left(b^2 + c^2 - 2bc\right) + \left(c^2 + a^2 - 2ac\right)\right] = 0$

or, $\frac{1}{2}\left[\left(a^2 + b^2 - 2ab\right) + \left(b^2 + c^2 - 2bc\right) + \left(c^2 + a^2 - 2ac\right)\right] = 0$

or, $(a-b)^2 + (b-c)^2 + (c-a)^2 = 0$ if $a \neq b \neq c$

Since $(a-b)^2 > 0, (b-c)^2 > 0 (c-a)^2 > 0$

Hence, $(a-b)^2 = 0 \Rightarrow a = b$

$(a-c)^2 = 0 \Rightarrow b = c$

$(c-a)^2 = 0 \Rightarrow c = a$

$\therefore a = b = c$ Hence Proved.

33. Image result for Prove that the ratio of the areas of two similar triangles is equal to the ratio of squares of their corresponding sides. Using the above, prove the following : In a Î ABC, XY is parallel to BC and it divides Î ABC into two parts of equal area. Prove that AB=2â â 12â .

Given: In a ΔABC, XY||BC and it divides ΔABC into two parts of equal area. i.e. Area($\triangle$BAC) = 2$\times$Area($\triangle$XAY)

To Prove: Prove that $\frac{\text{BX}}{\text{AB}} = \frac{\sqrt{2}-1}{\sqrt{2}}$.

Proof: Consider $\triangle BAC$ and $\triangle XAY$,

$\angle BAC = \angle XAY$ [common]

$\angle ABC = \angle AXY$ [corresponding angles]

$\Rightarrow \triangle BAC \sim \triangle XAY$ [By AA similarity rule]

We know that the ratio of the areas of two similar triangles is equal to the ratio of squares of their corresponding sides.

i.e. $\frac{area \Delta BAC}{area \Delta XAY} = (\frac{AB}{AX})^2$

$\frac{2 \times area \Delta XAY}{area \Delta XAY} = (\frac{AB}{AX})^2$ [Given]

2 = $(\frac{AB}{AX})^2$

$\Rightarrow \frac{AB}{AX} = \sqrt{2}$

$\Rightarrow \frac{AX}{AB} = \frac{1}{\sqrt{2}}$

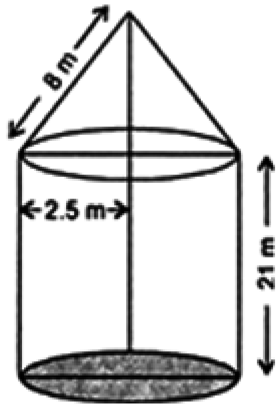
$\Rightarrow \frac{AB - BX}{AB} = \frac{1}{\sqrt{2}}$

$\Rightarrow 1 - \frac{BX}{AB} = \frac{1}{\sqrt{2}}$

$\Rightarrow \frac{\sqrt{2}-1}{\sqrt{2}} = \frac{BX}{AB}$

$\Rightarrow \frac{BX}{AB} = \frac{\sqrt{2}-1}{\sqrt{2}}$

Hence proved.

34. 

Radius of cylinder, r = 2.5 m, height of cylinder, h = 21 m, slant height of cone, l = 8 m

Total surface area of rocket = Curved of cylinder + Area of base + Curved surface area of cone

= $2\pi rh + \pi r^2 + \pi rl$

= $\pi r(2h + r + l)$

= $\frac{22}{7} \times 2.5\,(2 \times 21 + 2.5 + 8)$

= $\frac{22}{7} \times 2.5 \times 52.5$

= 412.5 m^2

OR

Surface area to colour = surface area of hemisphere + curved surface area of cone

Diameter of hemisphere = 3.5 cm

So radius of hemispherical portion of the lattu = r = $\frac{3.5}{2}$cm = 1.75

r = Radius of the concial portion = $\frac{3.5}{2}$ = 1.75

Height of the conical portion = height of top - radius of hemisphere = 5 - 1.75 = 3.25 cm

Let I be the slant height of the conical part. Then,

$l^2 = h^2 + r^2$

$l^2 = (3.25)^2 + (1.75)^2$

$\Rightarrow l^2 = 10.5625 + 3.0625$

$\Rightarrow l^2 = 13.625$

$\Rightarrow l = \sqrt{13.625}$

$\Rightarrow l = 3.69$

Let S be the total surface area of the top. Then,

$S = 2\pi r^2 + \pi rl$

$\Rightarrow \quad S = \pi r(2r + l)$

$\Rightarrow S = \frac{22}{7} \times 1.75(2 \times 1.75 + 3.7)$

$= 5.5(3.5 + 3.7)$

$= 5.5(7.2)$

$= 39.6\ cm^2$

35. We may observe from the given data that maximum class frequency is 40 belonging to 1500 - 2000 interval.

Class size (h) = 500

Mode = l + $\frac{f-f_1}{2f-f_1-f_2} \times$ h

Lower limit $(l)$of modal class = 1500

Frequency (f) of modal class = 40

Frequency (f_1) of class preceding modal class = 24

Frequency (f_2) of class succeeding modal class = 33

mode = 1500 + $\frac{40-24}{2\times 40-24-33} \times$ 500

= 1500 + $\frac{16}{80-57} \times$ 500

= 1500 + 347.826

= 1847.826 ≈ 1847.83

Expenditure (in ₹.)	Number of families f_i	x_i	$d_i = x_i - 2750$	u_i	u_if_i
1000-1500	24	1250	-1500	-3	-72
1500-2000	40	1750	-1000	-2	-80
2000-2500	33	2250	-500	-1	-33
2500-3000	28	2750=a	0	0	0
3000-3500	30	3250	500	1	30
3500-4000	22	3750	1000	2	44
4000-4500	16	4250	1500	3	48
4500-5000	7	4750	2000	4	28
	Σf_i = 200				$\Sigma f_i d_i$ = - 35

Mean $\overline{x} = a + \frac{\Sigma f_i d_i}{\Sigma f_i} \times$ h

$\overline{x}$ = 2750 + $\frac{-35}{200} \times$ 500

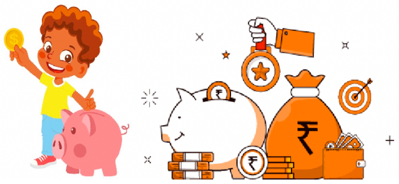
$\overline{x}$ = 2750 - 87.5

$\overline{x}$ = 2662.5

Section E

36. **Read the text carefully and answer the questions:**

Saving money is a good habit and it should be inculcated in children from the beginning. Mrs. Pushpa brought a piggy bank for her child Akshar. He puts one five-rupee coin of his savings in the piggy bank on the first day. He increases his savings by one five-rupee coin daily.



(i) Child's Day wise are,

$\underbrace{5}_{1\text{ coin}}, \underbrace{10}_{2\text{ coins}}, \underbrace{15}_{3\text{ coins}}, \underbrace{20}_{4\text{ coins}}, \underbrace{25}_{5\text{ coins}}$ .... to $\underbrace{n\text{ days}}_{n\text{ coins}}$

We can have at most 190 coins

i.e., 1 + 2 + 3 + 4 + 5 + ... to n term = 190

$\Rightarrow \frac{n}{2}[2 \times 1 + (n-1)1]$ = 190

$\Rightarrow$ n(n + 1) = 380 $\Rightarrow$ n^2 + n - 380 = 0

$\Rightarrow$ (n + 20)(n − 19) = 0 $\Rightarrow$ (n + 20)(n - 19) = 0

$\Rightarrow$ n = −20 or n = 19 $\Rightarrow$ n = -20 or n = 19

But number of coins cannot be negative

$\therefore$ n = 19 (rejecting n = -20)

So, number of days = 19

(ii) Total money she saved = 5 + 10 + 15 + 20 + ... = 5 + 10 + 15 + 20 + ... upto 19 terms

= $\frac{19}{2}$[2 × 5 + (19 - 1)5]

$= \frac{19}{2}[100] = \frac{1900}{2} = 950$

and total money she shaved = ₹950

OR

Number of coins in piggy bank on 15th day

$S_n = \frac{n}{2}[2a + (n-1)d]$

$\Rightarrow S_{15} = \frac{15}{2}[2 \times 5 + (15-1) \times 5]$

$\Rightarrow S_{15} = \frac{15}{2}$ [2 + 14]

$\Rightarrow$ S_{15} = 120

So, there are 120 coins on 15th day.

(iii) Money saved in 10 days

$S_n = \frac{n}{2}[2a + (n-1)d]$

$\Rightarrow S_{10} = \frac{10}{2}[2 \times 5 + (10-1) \times 5]$

$\Rightarrow S_{10}$ = 5[10 + 45}]

$\Rightarrow$ S_{10} = 275

Money saved in 10 days = ₹275

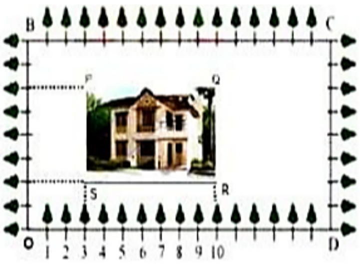
37. **Read the text carefully and answer the questions:**

Using Cartesian Coordinates we mark a point on a graph by how far along and how far up it is.

The left-right (horizontal) direction is commonly called X-axis.

The up-down (vertical) direction is commonly called Y-axis.

In Green Park, New Delhi Suresh is having a rectangular plot ABCD as shown in the following figure. Sapling of Gulmohar is planted on the boundary at a distance of 1 m from each other. In the plot, Suresh builds his house in the rectangular area PQRS. In the remaining part of plot, Suresh wants to plant grass.



(i)

Middle point of QS = $\left(\frac{10+3}{2}, \frac{6+2}{2}\right)$

= (6.5, 4)

(ii) Length = RS = $\sqrt{(10-3)^2 + (2-2)^2}$

RS = $\sqrt{7^2 + 0}$

RS = 7 m

Breadth = RQ = $\sqrt{(10-10)^2 + (2-6)^2}$

$= \sqrt{0 + 16}$

= 4 m

OR

Diagonal = $\sqrt{l^2 + b^2}$

$= \sqrt{7^2 + 4^2}$

$= \sqrt{49 + 16}$

$= \sqrt{65}$

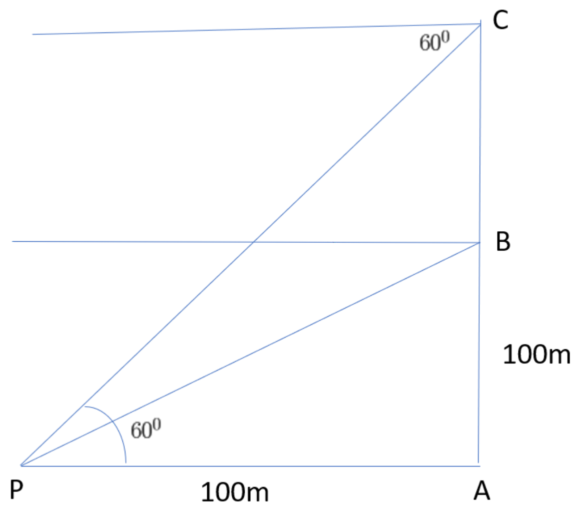
(iii) Area of rectangle = l $\times$ b

= 7 $\times$ 4

= 28 m^2

38. **Read the text carefully and answer the questions:**

A hot air balloon is rising vertically from a point A on the ground which is at distance of 100m from a car parked at a point P on the ground. Amar, who is riding the balloon, observes that it took him 15 seconds to reach a point B which he estimated to be equal to the horizontal distance of his starting point from the car parked at P.



(i) The angle of depression from the balloon at a point B to the car at point P.

In $\triangle$APB

tan B = $\frac{AB}{AP} = \frac{100}{100} = 1$

$\Rightarrow$ tan B = 1

$\Rightarrow$ tan B = tan 45^o

$\Rightarrow$ B = 45^o

(ii) The speed of the balloon is

Speed = $\frac{\text{Distance}}{\text{Time}}$

$\Rightarrow$ Speed = $\frac{100}{15} = \frac{25}{3}$ = 6.6 m/sec

OR

The vertical distance travelled by the balloon when angle of depression is 60^o.

In $\triangle$APC

Let BC = x

$\tan 60^\circ = \frac{AC}{AP} = \frac{AB+x}{100}$

$\Rightarrow \sqrt{3} = \frac{100+x}{100}$

$\Rightarrow 100\sqrt{3} - 100 = x$

$\Rightarrow$ x = $100(\sqrt{3} - 1)$

$\Rightarrow$ x = 73.21 m

(iii)The total time taken by the balloon to reach the point C from ground.

Time = $\frac{Distance}{\text{Speed}}$

$\Rightarrow T = \frac{100(\sqrt{3}-1)}{\frac{25}{3}}$

$\Rightarrow T = 12(\sqrt{3} - 1)$ = 8.78 sec